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Kolmogorov Dispersion for Turbulence in Porous Media: A Conjecture

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Abstract: We will utilise the self-avoiding walk (SAW) mapping of the vortex line conformations in turbulence to get the Kolmogorov scale dependence of energy dispersion from SAW statistics, and the knowledge of the disordered fractal geometries on the SAW statistics. These will give us the Kolmogorov energy dispersion exponent value for turbulence in porous media in terms of the size exponent for polymers in the same. We argue that the exponent value will be somewhat less than 5/3 for turbulence in porous media.

The turbulent flow of a fluid is a well known stochastic process with dispersion, where the energy transfers occur in various modes corresponding to different length scales. Based on the general properties of the Navier-Stokes equation of hydrodynamics and dimensional analysis, Kolmogorov obtained the energy spectrum E_k in the steady state of a fully grown turbulence as

$$E_k \sim k^{-\alpha}; \quad \alpha = 5/3, \tag{1}$$

in dimension $d = 3$, for an intermediate well-spread "inertial range" of the wave vector k [1].

This dispersion exponent α has been identified as the inverse Flory exponent ν for linear polymers or of SAWs [2]. In three dimensions, if the vortex lines are assumed not to cross (as in the absence of viscosity), the vortex line conformations can be modelled by those for SAWs. Hence the spatial distribution of energy density $E(r)$ can be obtained from the SAW pair correlation $g(r)$ (at a distance r), and the Fourier transform will then give

$$E_k \sim g_k. \tag{2}$$

The Fourier transform $g_k = g(ka, N)$ can be taken to be a function of the number of steps N of the SAW (each step size being a) and of the dimensionless number ka. Since the SAW end-to-end distance average R_N grows as aN^ν, where ν denotes the size

exponent, R_N remains invariant under the scale transformation $a \to al^\nu$ and $N \to N/l$. Since the pair correlation comes from the scatterings of $N-1$ other steps or monomers, it scales with N and hence

$$g(ka, N) = lg(kal^\nu, N/l). \tag{3}$$

Choosing $N/l = 1$, one gets $g_k = N\tilde{g}(kaN^\nu)$. Assuming now that $\tilde{g}(x) \sim x^{-\alpha}$, one gets

$$g_k \sim k^{-\alpha}N^{1-\nu\alpha} \sim k^{-\alpha}, \quad \text{if} \quad \alpha = 1/\nu. \tag{4}$$

This then, together with eqn. (2), gives eqn. (1) with $\alpha = 1/\nu$.

It is well known that the Flory estimate [3] for the polymer size exponent ν is

$$\nu = 3/(2+d). \tag{5}$$

This comes from the minimization of the free energy $f(R)$ of a chain of linear size (end-to-end distance) R: $f = f_e + f_r$ where $f_e \sim R^2/N$ denotes the Gaussian chain estimate of the elastic part and $f_r \sim R^d c^2 \sim N^2/R^d$ denotes monomer-monomer repulsive part of the free energy ($c = N/R^d$ denoting the monomer concentration). The minimisation of this f with respect to R gives $R \sim N^\nu$, where this Flory estimate of the ν is given by (5).

In $d = 3$, one gets $\nu = 3/5$ and hence $\alpha = 5/3$ as Kolmogorov obtained [2]. In $d = 2$, such a mapping (of the vortex lines to SAWs) does not exist and hence the correspondence between the Kolmogorov exponent α and the polymer/SAW size exponent ν (which of course is meaningful for the SAWs) can not be checked. At $d = 4$, of course, turbulence has not been studied.

However, checking in different (intermediate) dimensions could perhaps be done. In view of the wide ranging studies [4] of turbulence in porous media, in connection with oil explorations etc, one can easily extend the above correspondence (between the Kolmogorov energy dispersion exponent α and the polymer size exponent ν) in porous media, which has been modelled very extensively and successfully by percolating fractals [5]. In fact, the conformational properties of the linear polymers or SAWs in porous media have been extensively studied recently and often the rigorous theoretical and experimental or extensive computational results do not comfortably match each other [6]. May be, the studies on the energy dispersion for turbulence in porous media, the corresponding Kolmogorov exponent in particular, can also help in major reconciliation for the polymer conformation studies in porous media.

Since the fractal dimension d_F of the porous rock is certainly less than that (d) of the embedding dimension ($d_F \simeq 2.5$ for percolation clusters in $d = 3$ [5]), following

eqn. (5), the Flory estimate ν_F is clearly higher than that ($\nu = 3/5$) in the normal media ($d = 3$). This immediately indicates that the Kolmogorov energy dispersion exponent α for turbulence in a porous medium will be considerably below its standard value of 5/3. All the theoretical and computer simulation results indicate convincingly [6] that the SAW size exponent ν_F on the percolation like fractals are larger than that (ν) on the corresponding Euclidean lattices (in the same embedding dimensions). This would clearly indicate a lower (than 5/3) Kolmogorov exponent (α) value for turbulence in ($d = 3$) porous media. A recent approximate renormalization group study [7] of turbulence in porous media, however, seems to suggest unchanged value for the Kolmogorov exponent α. In view of this, rigorous theoretical as well as experimental studies on turbulence in porous media are needed to settle the issue.

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